UNDERSTANDING CIRCLE THEOREMS-PART ONE.

Common terms:

(a) **ARC-** Any portion of a circumference of a circle.

(b) **CHORD-** A line that crosses a circle from one point to another. If this chord passes through the centre then it is referred to as a diameter

(c) **A TANGENT-** A line that touches a circle at only one point.

Theorem 1.

The angle subtended at the centre of a circle is twice the angle subtended at the circumference by the same arc.

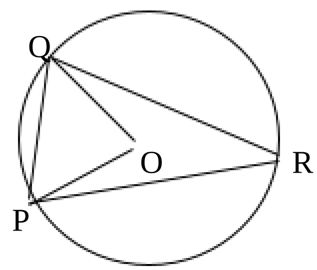
Theorem 2.

Angles subtended by an arc in the same segment of a circle are equal.

Example 1.

Given PQO = 65^0

Find QRP



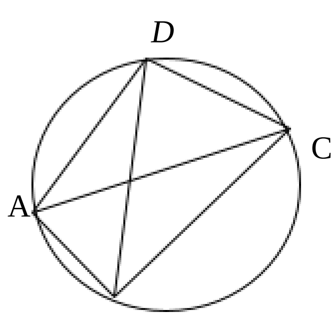
Triangle OQP is isosceles (OP = OQ, the radii)

:. OPQ = 65^0 :. QOP = $180^0 - (65^0 + 65^0)$ (angle sum of a triangle)

= 50^0

:. QRP = 25^0 (half of angle at the centre).

Example2.



Given that angle BDC = 78^0 and DCA = 56^0.

Find angles BAC and DBA.

B

Solution: BAC = BDC = 78^0. (both subtended by arc BC)

DBA = DCA = 56^0. (both subtended by arc AD)

Theorem3.

The opposite angles in a cyclic quadrilateral add up to 180^0 (the angles are supplementary).

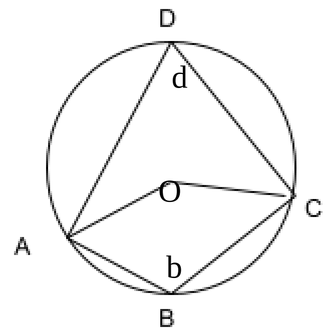
ABCD is a **cyclic quadrilateral** because all its vertices touch the circumference of the circle.(ABCO is not cyclic because O is not at the circumference).

Proof:

OA and OC are radii.

Let angle ADC = d

and angle ABC = b



AOC obtuse = 2d (angles at the centre)

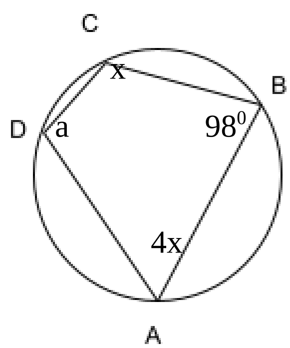
AOC reflex = 2b (angles at the centre)

:. $2d + 2b = 360^0$ (angles at a point)

:. $d + b = 180^0$ as required

Example3.

Find *a* and *x*



CDA = a, ABC = 98^0 , DCB = x^0 ,DAB = $4x^0$

$a = 180^0 - 98^0$ (opposite angles of a cyclic quadrilateral)

:. $a = 82^0$

$x + 4x = 180^0$ (opposite angles of a cyclic quadrilateral

$5x = 180^0$

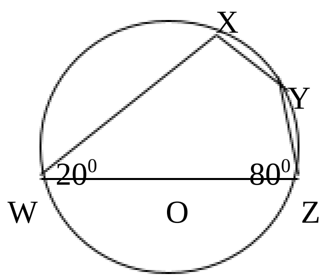
$x = 72^0$

Exercise. 1.ABCD is a quadrilateral inscribed in circle, centre O, and AD is a diameter of the circle. If angle CDB = 46^0 and ADB = 31^0. Calculate

(a) the angle ABC (b) the angle BCD (c) the angle BAD.

2. A circle has a radius of 155mm .AB is a chord of this circle which is 275mm long. What angle does AB subtend at the circumference of the circle.

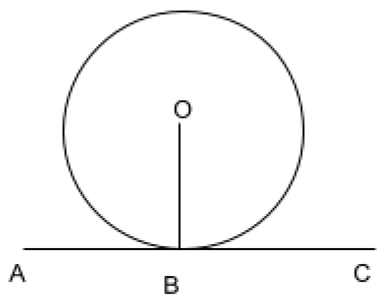
3. Given angle XWZ = 20^0 , angle WZY = 80^0 and O is the centre of the circle
   (a) Find angle WXY
   (b) Show that WY bisects XWZ



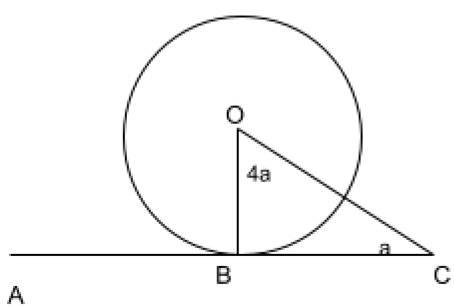
<u>Theorem 4.</u>

The angle between a tangent and the radius drawn to the point of contact is 90^0

Line ABC is a tangent and angle ABO = 90^0



Example 4. Find the angle BCO and angle BOC.



$$4a + a + 90^0 = 180^0$$

$$5a = 90^0$$

$$a = 18^0.$$

Angle BCO = 18^0 and BOC = 4 x 18^0 = 72^0.

UNDERSTANDING CIRCLE THEOREMS –PART TWO.

Theorem 5.

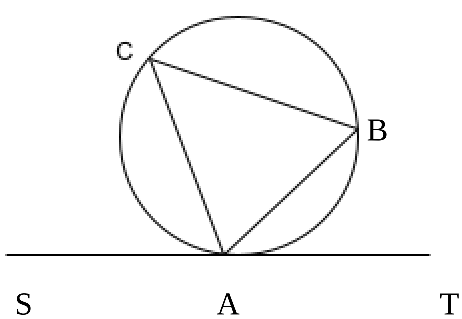
The tangents to a circle originating from a common point are equal in length.

Theorem 6.

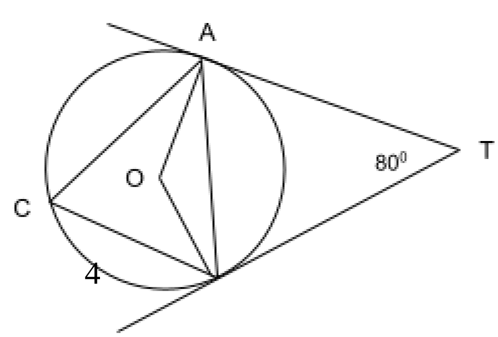
The Alternate segment theorem.

The angle between a tangent and chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

angle TAB = angle BCA and angle SAC = angle CBA



Example 1.



B

a) △ TBA is isosceles (TA = TB) ,angle TAB = angle TBA.

:. TBA = ½ (180 – 80)

$= 50^0$

b) $OBT = 90^0$ (tangent and radius)

$OBA = 90^0 – 50^0$

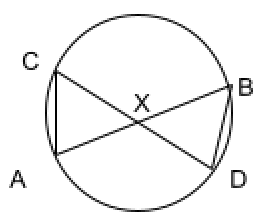
$= 40^0$.

c) ACB = ABT (alternate segment theorem)

$ACB = 50^0$

Theorem 7.

Intersecting chords theorem



AX.BX = CX.DX

Proof:

In triangles AXC and BXD:

Angle ACX = angle DBX (same segment)

Angle CAX = angle BDX (same segment)

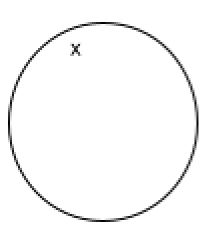
:. The triangles AXC and BXD are similar.

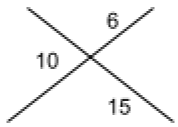
$$\frac{AX}{DX} = \frac{CX}{BX}$$

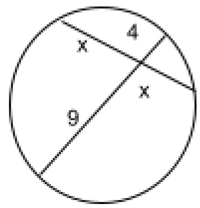
Thus **AX.BX = CX.DX**

Exercise.

Find x



(a) 

(b) 

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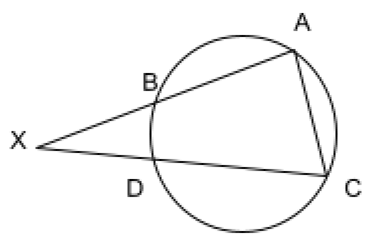
Theorem 8. **The intersecting secants theorem.**



Using triangles BXD and AXC;

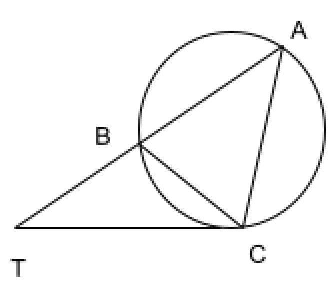
Angle XAC = angle XDB, angle XCA =angle XBD.(Figure BACD is a cyclic quadrilateral).Thus triangles AXC and BXD have equal angles and are similar.

$$\frac{AX}{DX} = \frac{CX}{BX}$$

Thus **AX.BX = CX.DX (This is the intersecting secants theorem)**

Theorem 9.

The secant/tangent theorem.

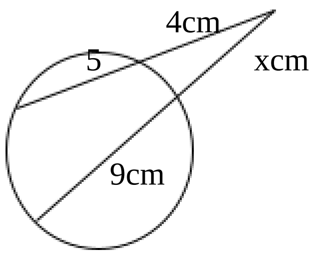


Angle BCT = angle BAC(alternate segment theorem). Triangles ATC and BTC share angle T and are similar triangles. (when triangles have two angles equal then they are similar.

In the triangles,

$\frac{AT}{TC} = \frac{TC}{BT}$;**AT.BT = TC^2(This is the secant/tangent theorem).**

Example 2.



Solution: $4 \times 9 = x.(9 + x)$

$36 = 9x + x^2.$

$x^2 + 9x – 36 = 0$

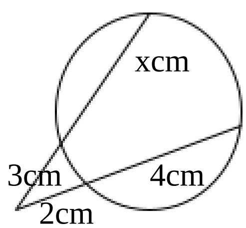
$x^2 + 12x – 3x – 36 = 0$

$x(x + 12) – 3(x + 12) = 0$

$(x – 3)(x + 12) = 0$; $x = 3$ or $x = ^{-}12$.

Since x cannot be negative then <u>x = 3 cm.</u>

Example 3.



Solution: $3 (3 + x) = 2 \times 6.$

$9 + 3x = 12.$

$3x = 3$

<u>$x = 1$cm</u>.

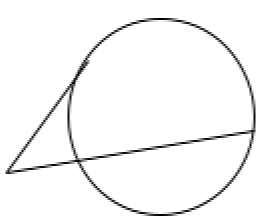
Exercise.

1. Two chords of a circle KL and MN intersect at X, and KL is produced to T. Given that KX = 6cm, XL = 4cm, MX = 8cm and LT = 8cm. calculate

a) NX

b) The length of the tangent from T to the circle

c) The ratio of the areas of △KXM to △LXN.

2. 

Find x.

6cm

xcm (x + 1)cm

3. Chords AB and BC of a circle are produced to meet outside the circle at T. a tangent is drawn from T to touch the circle at E. Given AB = 5cm, BT = 4cm and DC = 9cm, Calculate

a) CT b) TE

c) the ratio of the areas of $\triangle$ ADT to $\triangle$ BCT

d) the ratio of the areas of $\triangle$ BET to $\triangle$ AET